Optimization of a Radiation-Cooled Thermionic Converter

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The electrical power output of a radiation-cooled thermionic converter with a fixed radiator area is maximized with respect to the anode work function and to the ratio of cathode area to radiator area. The results show that, at high cathode temperature, it is important to cool the anode at the expense of power-generating area, whereas at low cathode temperature, the reverse is true. In addition, the results show that, at high cathode temperature, the optimum anode work function and temperature are equal to about 75% of the corresponding cathode work function and temperature.

Introduction

THE concept of a nuclear-thermionic space power supply recently has aroused considerable interest. The basic components of such a power supply are the thermionic converters, the nuclear reactor that heats the converters, and the radiator that rejects the unconverted heat. The converters may be an integral part of the reactor fuel elements, in which case a liquid metal must be used to transfer waste heat from the converters to an external radiator. Several systems of this type have been considered.[1–4] A different approach is to attach the converters to the surface of a cylindrical reactor and let the anodes radiate waste heat directly.[5] The latter system, which is discussed in this paper, differs from the former in that the radiator area is determined by the reactor surface area. In this case, the electrical power output per unit area of reactor surface (the *specific* power output) can be used as a figure of merit. The specific power output will, in general, be less in the case of a radiation-cooled space powerplant than in the case of a convection-cooled powerplant because the anodes of the converters must operate at high temperatures in order to reject the waste heat. For example, if the converters deliver 5 w/cm² at an efficiency of 10%, then about 45 w/cm² must be rejected. Assuming a blackbody radiator, the temperature required to radiate 45 w/cm² is about 1700°K. At this high anode temperature, back emission from the anode can reduce the power output of a converter to a fraction of what it would be in the case of a cold anode (zero back emission).

With a fixed radiator area, a basic question is to what degree the specific power output of a system can be increased by sacrificing electron-emitting area for a lower anode temperature and less back emission. (This situation might be achieved by using thermal shields between portions of the reactor surface and the anode.[6] The subject of this paper is an analysis aimed at answering this question. In particular, the problem of maximizing the specific power output of a converter subject to a thermal radiation boundary condition is formulated, and the numerical results obtained for a range of cathode temperatures and work functions are presented. These results indicate that a significant specific power output can be generated by a radiation-cooled system and that an increase in the specific power output of such a system can be obtained by thermally shielding a sizeable fraction of the reactor surface.

Theory

The formulation of the problem is based on a relatively simple model of a thermionic converter in which it is assumed that space charge is completely neutralized and that the net electric current is given by

$$I = (J_c - J_a)S_c$$

where J_c and J_a are the cathode and anode emission current densities and S_c is the effective cathode area. For a voltage V applied to the anode, J_c and J_a are assumed to follow the Richardson-Dushman emission equation as follows:

$$\begin{aligned} J_c &= A_cT_c^2 \exp\{-e(\varphi_a + V)/kT_c\} && V \leqq \varphi_c - \varphi_a \\ &= A_cT_c^2 \exp\{-e\varphi_c/kT_c\} && V \geqq \varphi_c - \varphi_a \\ J_a &= A_aT_a^2 \exp\{-e(\varphi_c - V)/kT_a\} && V \geqq \varphi_c - \varphi_a \\ &= A_aT_a^2 \exp\{-e\varphi_a/kT_a\} && V \leqq \varphi_c - \varphi_a \end{aligned}$$

where T is the absolute temperature, e the electronic charge, k the Boltzmann constant, φ the work function, and A the emission constant.

Figures 1a and 1b show ideal current-voltage and power-voltage characteristics, respectively, corresponding to the simple model just outlined for the case in which the intrinsic anode work function φ_a is less than the optimum. Note that in this case maximum power does not occur at an applied voltage $V = \varphi_c - \varphi_a$, but at $V = \varphi_c - V_a$, where V_a is defined as the effective anode work function. Figure 2 is a potential diagram showing how φ_c, φ_a, V_a, and V are related in the case of positive ion sheaths at both electrodes. (For an electron-rich sheath, φ_c would refer to the sum of the work function and the sheath potential.) Figures 3a and 3b show ideal current-voltage and power-voltage characteristics, respectively, for the special case in which $V_a = \varphi_a$. Figure 3b shows that although maximum power now occurs at

$$V = \varphi_c - \varphi_a$$

it is just equal to the maximum power in the general case (cf., Fig. 1b). Thus the total electrical power output of the converter under consideration can be written as

$$P_0 = (J_c - J_a)(\varphi_c - V_a)S_c = f(J_c - J_a)(\varphi_c - V_a)S_r \quad (1)$$
$$V_a \geqq \varphi_a$$

where S_r is the area of the reactor covered by a converter and f is the ratio (S_c/S_r). In this case the back current from the anode is

$$J_a = A_aT_a^2 \exp\{-eV_a/kT_a\}$$

Part of the total electrical power output is dissipated in the electrical lead required to connect the load to the device; for convenience, the *useful* power output is written as

$$P_0 = f(J_c - J_a)(\varphi_c - V_a)(1 - \Delta)S_r$$

where

$$\Delta = (J_c - J_a)S_cR_l/(\varphi_c - V_a)$$

The lead resistance R_l can be chosen so that $\Delta \ll 1$, in which case the useful power output is given to a good approxima-

Received by ARS November 16, 1962.

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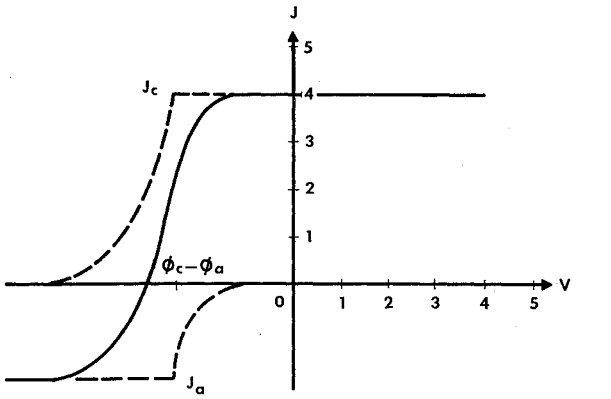

Fig. 1a Ideal current-voltage characteristic for $\varphi_a < V_a$

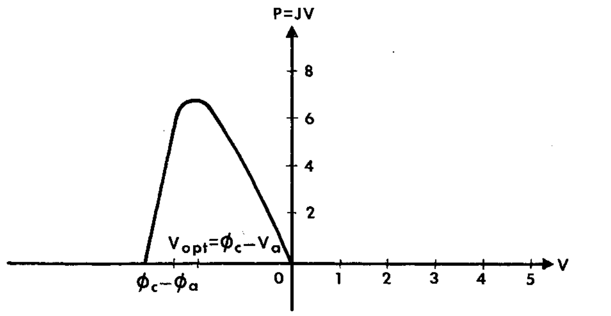

Fig. 1b Ideal power-voltage characteristic for $\varphi_a < V_a$

tion by Eq. (1). Note that the lead loss can be taken into account if necessary by replacing f by $\bar{f}$, where $\bar{f} = f\,(1 - \Delta)$.

The anode temperature is related to the cathode temperature by a steady-state heat balance equation. The heat flow into the anode consists of the following:

P_1 = electron heating of the anode
P_2 = radiant heating of the anode
P_3 = heat conduction in the electrical lead, if any, connecting cathode to anode

The heat flow out of the anode consists of P_4 = radiant cooling of the backside of the anode.

For the application being considered, it is assumed that 1) the heat conducted from the cathode in the electrical lead is dissipated before reaching the anode; 2) the heat conducted through the gas is small compared to the heat transferred by radiation; and 3) the heat conducted from the anode in the electrical lead is small compared to the heat radiated from the anodes. Then the heat balance equation is

$$0 = P_1 + P_2 - P_4$$

In accordance with the model chosen, the electron heating of the anode is given by

$$P_1 = f\{J_c(V_a + 2\theta_c) - J_a(V_a + 2\theta_a)\}S_r$$

where $\theta = kT/e$. The heating of the anode due to transfer of heat by radiation between the cathode and anode is given by

$$P_2 = f\alpha(\theta_c^4 - \theta_a^4)S_r$$

where $\alpha = \sigma(e/k)^4(\epsilon_c^{-1} + \epsilon_a^{-1} - 1)^{-1}$, with σ being the Stefan-Boltzmann constant and ϵ the relative total emissivity. This is based on the assumptions of infinite, parallel plane geometry and of perfect thermal shielding of that part of the reactor surface not covered by cathode.

The cooling due to radiation of heat from the back of the anode is given by

$$P_4 = \beta\theta_a^4S_r$$

where $\beta = \epsilon_a'\sigma(e/k)^4$, and an isothermal radiator with a surface area per diode equal to S_r has been assumed.

Hence, the problem is to maximize the specific power output, given by

$$P_s[V_a, f, T_a(V_a, f)] = f(J_c - J_a)(\varphi_c - V_a)$$

with respect to V_a and f, subject to the constraint

$$0 = f\alpha(\theta_c^4 - \theta_a^4) - \beta\theta_a^4 + f[J_c(V_a + 2\theta_c) - J_a(V_a + 2\theta_a)]$$

By making the substitutions $x = T_a/T_c$ and $y = eV_a/kT_a$, these equations can be written as

$$P_s(x, y) = f(1 - \xi x^2 e^{\gamma - y})(\gamma - xy)J_c\theta_c \tag{2}$$

$$0 = \alpha'(1 - x^4) - (\beta'x^4/f) + 2 - (y + 2)\xi x^3 e^{\gamma - y} \tag{3}$$

where $\xi = A_a/A_c$, $\gamma = \varphi_c/\theta_c$, $\alpha' = \alpha\theta_c^3/J_c$, and $\beta' = \beta\theta_c^3/J_c$. By solving Eq. (3) for f in terms of x and y and substituting the result in Eq. (2), one has the following expression for P_s in terms of the independent variables x and y:

$$P_s(x, y) = \frac{x^4(1 - \xi x^2 e^{\gamma - y})(\gamma - yx)}{\alpha' + 2 + yx - (y + 2)\xi x^3 e^{\gamma - y} - \alpha' x^4}\,\beta\theta_c^4 \tag{4}$$

In order for P_s to be maximum, it is required that $\partial P_s/\partial x =$

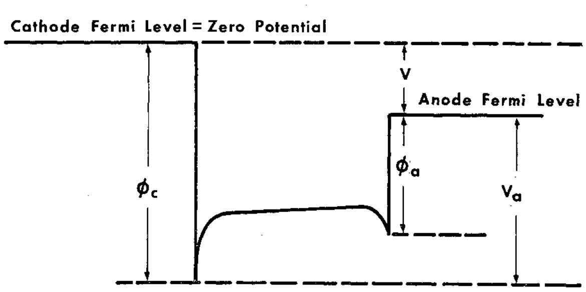

Fig. 2 Potential diagram

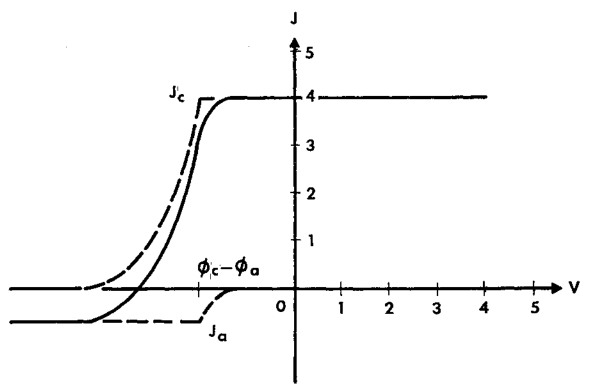

Fig. 3a Ideal current-voltage characteristic for $\varphi_a = V_a$

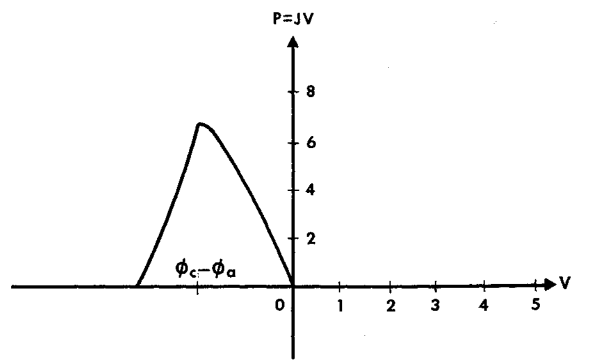

Fig. 3b Ideal power-voltage characteristic for $\varphi_a = V_a$

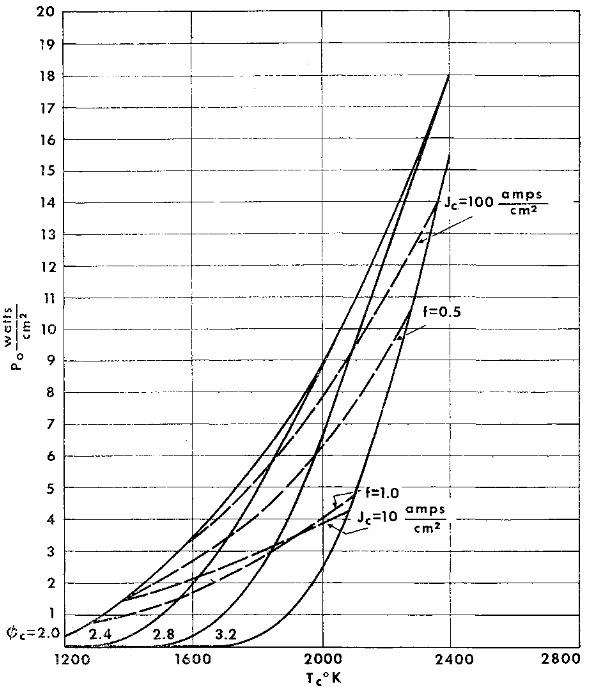

Fig. 4 Maximum specific power

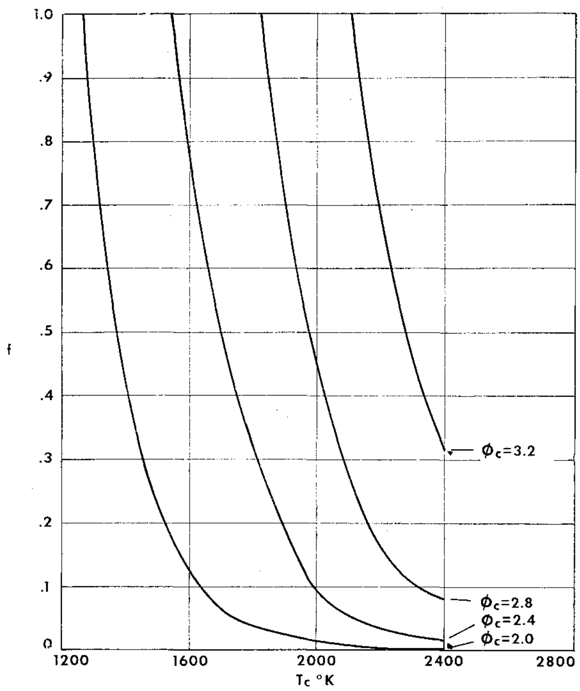

Fig. 5 Optimum area ratio

0 and $\partial P_s/\partial y = 0$. Imposing this requirement on Eq. (4) yields the following pair of equations:

$$0 = \sum_{n=0}^{6} C_n(y)x^n \tag{5}$$

$$0 = \sum_{n=0}^{7} D_n(y)x^n \tag{6}$$

where the coefficients C_n and D_n are given by

$$\begin{aligned}
C_0 &= \gamma + \alpha' + 2\\
C_1 &= -\gamma(\alpha' + 2)\xi e^{\gamma-y}\\
C_2 &= (\alpha' + 2)(y - 1)\xi e^{\gamma-y}\\
C_3 &= -2(y + 1)\xi e^{\gamma-y}\\
C_4 &= \gamma\xi e^{2(\gamma-y)} - \alpha'\\
C_5 &= 2\xi^2 e^{2(\gamma-y)} + \gamma\alpha'\xi e^{\gamma-y}\\
C_6 &= -\alpha'(y - 1)\xi e^{\gamma-y}\\
D_0 &= -4\gamma(\alpha' + 2)\\
D_1 &= [5(\alpha' + 2) - 3\gamma]y\\
D_2 &= 6\gamma(\alpha' + 2)\xi e^{\gamma-y} + 4y^2\\
D_3 &= \{[6\gamma - 7(\alpha' + 2)]y + 2\gamma\}\xi e^{\gamma-y}\\
D_4 &= -4y(2y + 1)\xi e^{\gamma-y}\\
D_5 &= -3\gamma(y + 2)\xi e^{\gamma-y} - \alpha' y\\
D_6 &= 4y(y + 2)\xi e^{2(\gamma-y)} - 2\gamma\alpha'\xi e^{\gamma-y}\\
D_7 &= 3\alpha'\xi\, y e^{\gamma-y}
\end{aligned}$$

The simultaneous solution of Eqs. (5) and (6) yields the optimum values of x and y. The optimum value of f then is obtained from Eq. (3).

Discussion of Results

In order to obtain a semiquantitive estimate of the maximum specific electrical power output of a nuclear-thermionic space power supply, some typical values were assigned to the parameters in Eqs. (5) and (6). The cathode and anode emission constants A_c and A_a were taken to be 120 amps/cm²-deg². The emissivities ϵ_c, ϵ_a, and ϵ_a' were taken to be 0.4, 0.8, and 0.9, respectively. Equations (5) and (6) were solved simultaneously to give optimum values of x and y for a range of cathode temperatures and work functions. The optimum value of f then was obtained from Eq. (3).

Using these optimum values of x, y, and f, the maximum specific electrical power output was computed from Eq. (2). Figure 4 shows how the maximum specific power output varies with cathode temperature and work function. The equi-current and equi-area-ratio lines are included for reference purposes. Note that, for cathode temperatures above 2200°K, the maximum specific power output is independent of cathode work function for the range of 2.0 to 2.8 v. Note also that only for optimum area ratios less than unity does the maximum power output become interesting. This means that an optimized system with an area ratio greater than unity, which can be obtained by inserting converters in radial holes in the reactor fuel, is not promising. As can be seen from Fig. 5, the optimum area ratio decreases monotonically with cathode temperature. This means that, at high cathode temperatures (high output power densities), it is important to cool the anode at the expense of power generating area, whereas at low cathode temperatures (low output power densities), the reverse is true.

Figure 6 shows the variation of the optimum effective anode work function with cathode temperature and work function. According to the simple converter model used for this analysis, the optimum output voltage is the difference between φ_c and V_a. Note that at the higher cathode temperatures the optimum value of V_a is about 75% of the cathode work function.

The optimum value of the anode temperature is limited to about 75% of the cathode temperature, as shown in Fig. 7. This is in agreement with the general result that is found for an imperfect Carnot engine that is cooled by radiation. If the efficiency of the imperfect Carnot engine is proportional to Carnot efficiency, then it can be shown that, in order to maximize the power output per unit area of radiator, the radiator temperature must be between 0.75 and 0.80 times the inlet temperature (see Appendix).

The efficiency of the conversion process is defined as the ratio of electrical power output to the total heat input to the cathode. In the steady state, the heat input to the cathode is equal to the heat dissipated from the cathode, which consists of the radiant cooling P_2, the lead conduction cooling P_3, and the electron cooling. The latter is given by

$$P_5 = f[J_c(\varphi_c + 2\theta_c) - J_a(\varphi_c + 2\theta_a)]S_r = P_0 + P_1$$

Thus the efficiency is given by

$$\eta = P_0/(P_2 + P_3 + P_5) = P_0/(P_0 + P_1 + P_2 + P_3)$$

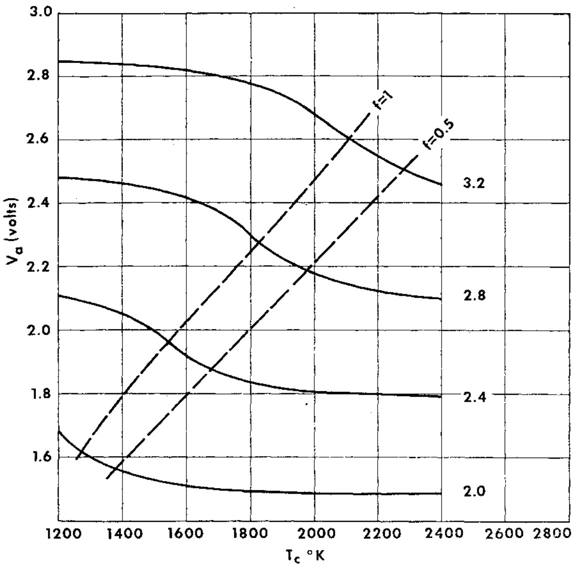

Fig. 6 Optimum anode work function

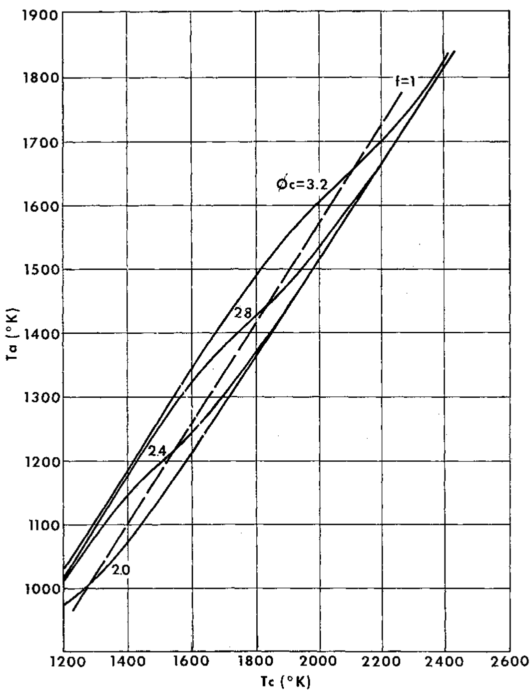

Fig. 7 Optimum anode temperature

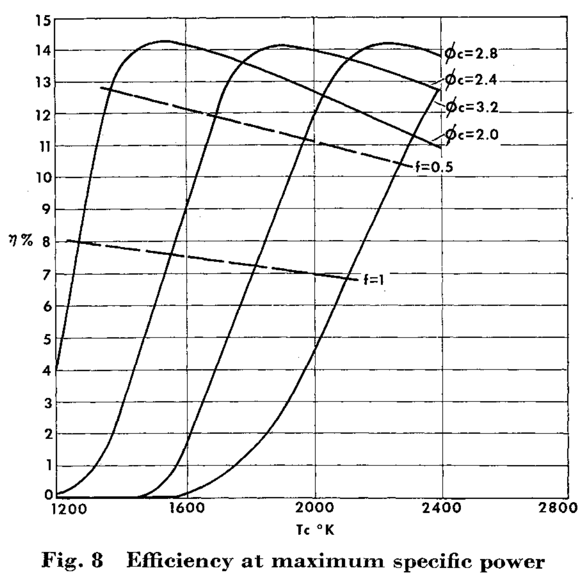

Fig. 8 Efficiency at maximum specific power

If radiation from the leads is neglected, P_3 is given by[7]

$$P_3 = \kappa(a/l)(T_c - T_0) - (J_c - J_a)^2 S_c\rho(l/a)/2$$

where κ is the thermal conductivity, ρ the electrical resistivity, and (l/a) the ratio of length to cross-sectional area. If an approximate form of the Wiedemann-Franz relation

$$\kappa\rho = (\pi^2k^2/6e^2)(T_c + T_0)$$

where T_0 is the heat sink temperature, is used, then the expression for P_3 can be written as follows:

$$P_3 = f\{\pi^2[1 - (\theta_0^2/\theta_c^2)]/6\Delta(\gamma - xy) - \Delta(\gamma - xy)/2\}(1 - \xi x^2e^{\gamma-y})J_c\theta_cS_r$$

For $\Delta \ll 1$, this reduces to

$$P_3 = \pi^2[1 - (\theta_0^2/\theta_c^2)]\,(1 - \xi x^2e^{\gamma-y})fJ_c\theta_cS_r/6\Delta(\gamma - xy)$$

Thus, the efficiency is given by the equation

$$\frac{1}{\eta} = 1 + \frac{P_1 + P_2 + P_3}{P_0} = 1 + \frac{\alpha'(1 - x^4) + xy + 2 - (y + 2)\xi x^3e^{\gamma-y}}{(1 - \xi x^2e^{\gamma-y})(\gamma - xy)} + \frac{\pi^2}{6\Delta}\,\frac{1 - (\theta_0^2/\theta_c^2)}{(\gamma - xy)^2}$$

The variation of the efficiency with cathode temperature and work function for the optimum values of x and y with $\Delta = 0.05$ and $\theta_0/\theta_c = \frac{1}{2}$ is shown in Fig. 8.

To illustrate how the foregoing results can be used to estimate the operating characteristics of a radiation-cooled nuclear thermionic space power supply, suppose that a cathode with a work function of 2.8 v were operating at 2000°K. According to Fig. 4, the maximum specific power output would be 6.5 w/cm². To get this power output, the following conditions would be required: $f = 0.45$ (Fig. 5); $V_a = 2.2$ v (Fig. 6); $T_a = 1560$°K (Fig. 7); $\eta = 11.8\%$ (Fig. 8). Since Carnot efficiency would be 22%, the overall efficiency of the power supply would be just slightly more than 50% of Carnot efficiency.

It is realized that the simple model on which the foregoing analysis is based does not take into account some of the physical processes that occur in the converter; consequently, the numerical results should not be regarded as quantitative results. Specifically, the present treatment neglects the effects of space charge and transport effects in the plasma. It should be emphasized that the main objective of this analysis was to obtain information about the qualitative behavior of a converter with a radiation-cooled anode. It is felt that modification of the model used in this analysis to include the effects just mentioned will not alter this qualitative behavior.

Summary and Conclusions

The power output of a radiation-cooled converter has been maximized with respect to the anode work function and to the ratio of cathode to radiator surface area. The results show that, at high cathode temperatures where the output power density is high, it is advantageous to cool the anode at the expense of power-generating area, whereas at low cathode temperatures where the output power density is low, the reverse is true. For the cases studied, the optimum anode work functions and temperatures were approximately 75% of the corresponding cathode work functions and temperatures, respectively, in the range $\gamma < 16$. A further result obtained from this analysis is that the specific power output is a monotonic decreasing function of the optimum area ratio; hence the area ratio should be made as small as possible within the other constraints of the problem.

Appendix

The efficiency, which is defined as the ratio of power output P_0 to power input P_i, is taken to be proportional to Carnot efficiency, i.e.,

$$P_0/P_i = a(1 - x) \qquad 0 < a \leq 1 \tag{A1}$$

The parameter x is the ratio of radiator temperature T_r to input temperature T_i. The radiated power P_r, which is equal to the difference between P_0 and P_i, is given by

$$P_r = P_0 - P_i = bx^4 \tag{A2}$$

where $b = A_r\, \epsilon\sigma T_i^4$, with A_r being the radiator area, ϵ the relative emissivity, and σ the Stefan-Boltzmann constant. Eliminating P_i from Eqs. (A1) and (A2) gives

$$\psi = -ax^4(1 - x)/[a(1 - x) - 1] \tag{A3}$$

where $\psi = P_0/b$ is proportional to the power output per unit area of radiator. To maximize ψ, it is required that $d\psi/dx = 0$. This yields the following equation for the optimum value of x:

$$0 = 4a(1 - x)^2 - 5(1 - x) + 1$$

Thus for $a = 1$, the optimum value of x is 0.75, whereas in limit $a \rightarrow 0$, it is 0.80. This means that, even for a very poor Carnot engine ($a < 1$) that is cooled by radiation, the optimum radiator temperature is not much different from that of a perfect Carnot engine.

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MAY 1963 AIAA JOURNAL VOL. 1, NO. 5

Nuclear Rocket Thrust Optimization Using Dynamic Programming

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The problem considered herein is that of maximizing the "burnout" velocity of a solid core nuclear rocket. The optimal thrust schedule has been shown, for a range of realistic parameter values, to differ from the well-known result for chemical systems because of the necessity to remove core fission product decay afterheat. This afterheat, which must be removed if the core is to remain intact, is a function of the reactor operating power schedule. The criterion of optimality is the maximization of the rocket final velocity under the constraints of a maximum and minimum allowable reactor power level corresponding to engine throttleable limits and a fixed initial propellant loading that is to be allocated either as useful thrust-producing propellant or as a core afterheat removal coolant yielding no useful thrust. The method of dynamic programming is used to perform the optimization. Numerical results are presented for an example problem that considers vertical, drag-free liftoff from a stationary earth with a uniform gravitational field. Means to relax all of these restrictions, along with other important extensions, are discussed. The effect of varying the problem parameters also is indicated, and implications thereof are pointed out.

Statement of the Problem

No doubt a lingering vestige of the chemical rocket propulsion era, it appears to be a common assumption that nuclear-powered rockets will perform at constant maximum power and thrust. In lieu of a better alternative, this certainly is a reasonable assumption. It is the purpose of this paper to examine that assumption and to determine whether or not a better alternative is available. The method of dynamic programming is well suited to answer just such a question.

Perhaps the definitive applications of the technique of dynamic programming to nuclear reactor systems optimization are those by Ash, Bellman, and Kalaba[1] and Kallay.[2, 3] The original works by Bellman, e.g., Refs. 4 and 5, remain the most readable, informative, and amusing introduction to the subject of dynamic programming. Further applications of dynamic programming to a wide range of optimization problems encountered in engineering are given by Kalaba.[6]

Received by ARS October 29, 1962. This paper was prepared under the sponsorship of the Douglas Aircraft Company Independent Research and Development Program, Account No. 81425-110, E.W.O. No. 51970. The writer counts it a pleasure to record that his stimulating introduction to matters programming, both Fortran and dynamic, was provided by N. Kallay. In a series of lectures given at the University of California at Los Angeles, R. E. Kalaba carefully nurtured both interests. His personal assistance in the preparation of this paper also is acknowledged gratefully.

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